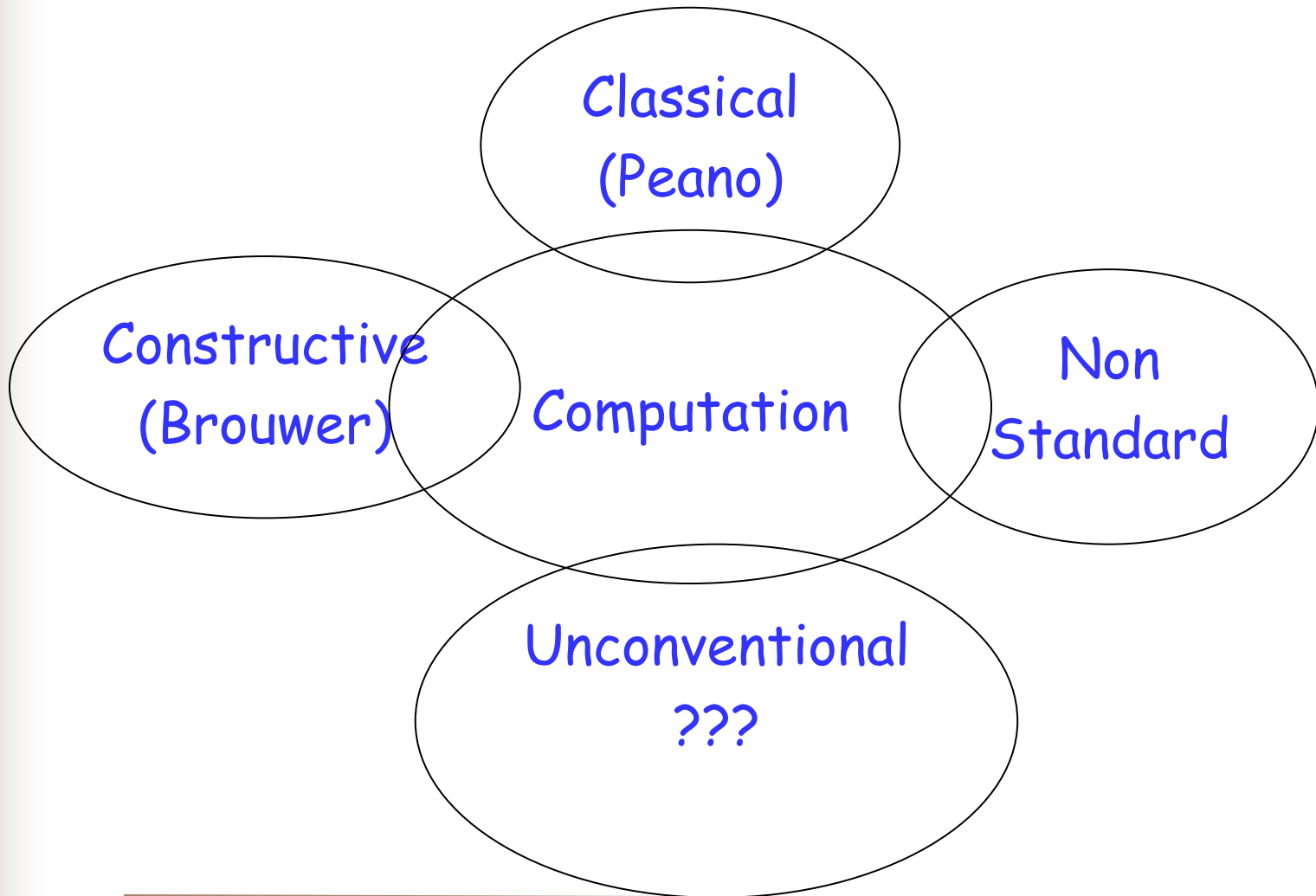


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**UNCONVENTIONAL  
MATHEMATICS**

# Unconventional mathematics ?



# Turing Machines

- **Church-Turing Thesis**
  - ✓ *Every function which would naturally be regarded as computable can be computed by a Turing Machine*
- **Physical CTT (PCTT)**
  - ✓ *every function that can be physically computed, can be computed by a Turing machine*
- **Strong Church-Turing Thesis (SCTT)**
  - ✓ *any 'reasonable' model of computation can be efficiently simulated on a probabilistic Turing machine [Bernstein, Vazirani, 1997]*

# Beyond the Turing Limit

- **Super-Turing Computation**
  - ✓ any computation that cannot be carried out by a Turing Machine as well as any computation carried out by a Turing Machine
- **Super-Turing computer**
  - ✓ any system or device capable of carrying out Super-Turing computation
- **Note: There are two possible points of view:**
  - ✓ Compute the TM non-computable problems
  - ✓ Treat the TM untreatable problems

# Beyond the Turing Limit

- Oracle Machines (o-machines) [Turing, 1939]
- Choice Machines (c-machines) [Turing, 1936]
- Unorganized Machines (u-machines) [Turing, ?]

# How to empower a Turing Machine?

- **Interaction with the external world**
- **Evolution/adaptation/learning**
- **Infinity: releasing restriction on boundedness of resources**

# Unconventional computation models

- Cellular Automata [Von Neumann]
  - ✓ Wolfram's New Kind of Science
- Interaction Machines (Ims) [Wegner, 1998]
  - ✓ allow inputs to be dynamically generated and require inputs to be represented by a potentially infinite stream
- Persistent Turing Machines [Goldin, 2000]
- *Site machines* [Van Leeuwen & Wiedermann, 2000]
  - ✓ communicate with the environment by sending and receiving messages through several input and output ports

# Unconventional computation models

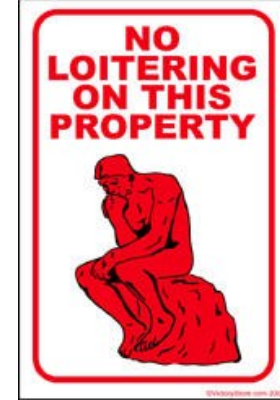
- Accelerating Turing Machines [Copeland, 1998]
  - ✓ programs are executed at an ever-accelerating rate
- Hypercomputation
  - ✓ infinite number of steps in a finite time
- Nanocomputation
- Analog computation
- Field computation
- Biological computation
  - ✓ DNA
  - ✓ Membrane
- Quantum computation



# Numbers and their representations

- Computation over reals: the Blum-Shub-Smale model
- Neural computing: trains of spikes, unary representation
- Membrane computing: spiking neural P systems
- A general question:

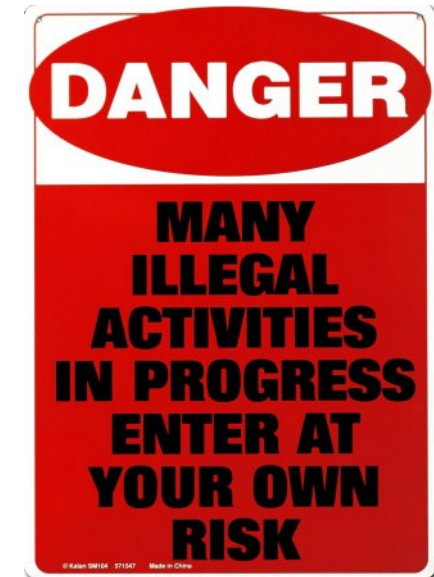
How information and numbers are encoded in  
"natural" systems ?



*“Every attempt to employ mathematical methods in the study of biological questions must be considered profoundly irrational and contrary to the spirit of biology.”*

*“If mathematical analysis should ever hold a prominent place in biology – an aberration which is happily almost impossible – it would occasion a rapid and widespread degeneration of that science.”*

**Auguste Comte, *Pilosophie Positive*, 1830**



# Biology vs mathematics

- **Mathematical biology**
  - ✓ use of mathematics to describe or model biological systems.
- **Biological mathematics**
  - ✓ study of mathematics as it occurs in biological systems.

# Formal world vs real world

- Formal models of
  - ✓ emergence
  - ✓ adaptive dynamics
  - ✓ evolutionof self-assembling, self-organizing, self-maintaining and self-replicating systems
- Understanding of computation and communication in the living nature
- Definition of abstract information structures and processes

## Some approaches

- Splicing systems [Head 1987]
- DNA Computing [Adleman 1994]
  
- BUT ...
  
- Information is not encoded only in the sequence
  - ✓ Structure
  - ✓ Networks
  - ✓ Long distance interactions ...
  - ✓ Stochasticity/randomness

# Unconventional Mathematics ?

➤ All of the above

OR

➤ Nothing of the above

???